# Color Theory

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## Contents

Different materials reflect different frequencies of light better than others, creating an imbalance in the light's spectral power distribution. Our eyes pick up certain frequencies in different intensities, and our brains combine these signals to create color vision.

We should first develop a (sort of) rigorous model on the set of colors and what "operations" you can do with them.

**Definition 0.1 (Color Space)**

Let $V$ be a topological vector space over $\mathbb{R}$.[a] A **color space** is an abelian group $(C, m)$, where
1. $C$ is a convex subset of $V$ containing 0.
2. $m$ is a continuous **color mixing** operator

$$m : C \times C \to C, m(c_1, c_2) \coloneqq \frac{1}{2}(c_1 + c_2) \tag{1}$$

which represents the new color $m(c_1, c_2)$ you get from mixing colors $c_1$ and $c_2$ in equal volume. It is closed under the convex set by definition and can be interpreted as the midpoint between two colors.

[a] We can think of $V$ as a set representing some idea that generalizes the concept of color, supporting scalar multiplication, inverse, etc.

One might ask what the dimension of $V$ is, and my answer is: we don't know and don't care. This is because eventually, we will be working only with $C$ and will be treating it as a group anyways. So a better representation using convex hulls is more practical.

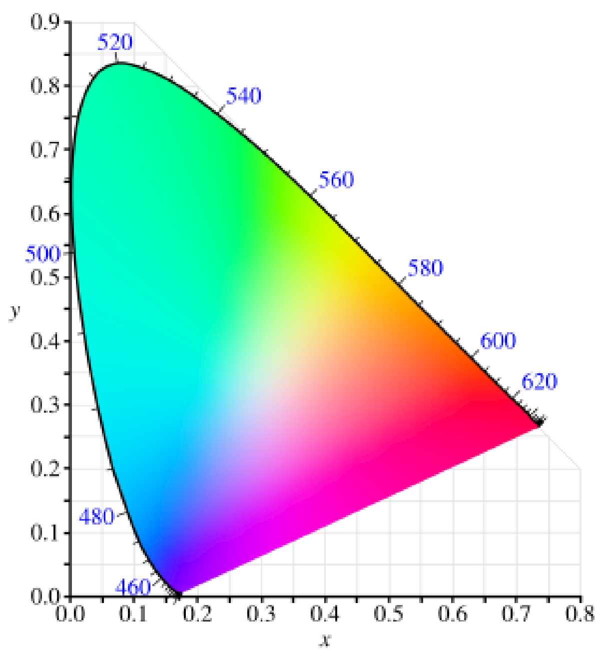



Figure 1: The CIE 1931 color space depicts the color space of all visible colors to the human eye. The white background is the vector space $V$, and the "horseshoe" shaped set is $C$. It is the most popular visual of color spaces. Note that in this visual, we treat these colors as living in 2 dimensions.

Note that this was quite an abstract definition, and we do this to construct a common interface between various color models. We have not stated what color each vector may represent (e.g. 0 is what color?), nor have we defined what *mixing* means (in terms of mixing paint or overlapping flashlight?). There are two broad categories that specify these.

**Definition 0.2 (Additive Color Space)**

In an **additive color space**, the

1. 0 vector is black.
2. $m(c_1, c_2)$ is the color you get when you shine two flashlights of colors $c_1$ and $c_2$—of equal intensity—into the same spot.

This is what you will probably use on a day-to-day basis. This is the standard for monitors specifications and visual design.

**Definition 0.3 (Subtractive Color Space)**

In a **subtractive color space**, the

1. 0 vector is white.
2. $m(c_1, c_2)$ is the color you get when you mix two paints of colors $c_1$ and $c_2$—of equal volume—into the same bucket.

This is the norm for printers.

Besides convexity, we don't really care about the vector space operations on $V$. We would like a more succinct representation of $C$.

**Definition 0.4 (Gamut)**

The **gamut** of a finite set of colors $P = \{c_i \in V\}_{i=1}^n$ is the convex hull of $P$. A set of points $P$ is said to **generate** a color space $C$ if the convex hull of $P$ is $C$.

**Definition 0.5 (Primary Colors)**

The **primary colors**, or **hues**, of a color space $C$ is any minimal[a] set of points $P$ that generates $C$.

[a]as in, we cannot generate the same space $C$ with a strict subset of $P$

It is generally accepted that we need a set of 3 primary colors to accurately model the color space of visible colors. We mainly believe it to be 3 because the human eye has 3 main color receptors, but this has no strict justification. Which three primary colors we select determines the model. We will look at RGB, RYB, and CMY, which are the three most popular ones.

**Definition 0.6 (Secondary Colors)**

Given a set of primary colors $P = \{p_1, p_2, p_3\}$ that generate a color space $(C, m)$, the **secondary colors** $S = \{s_1, s_2, s_3\}$ are defined

$$s_1 = m(p_2, p_3) \tag{2}$$
$$s_2 = m(p_1, p_3) \tag{3}$$
$$s_3 = m(p_1, p_2) \tag{4}$$

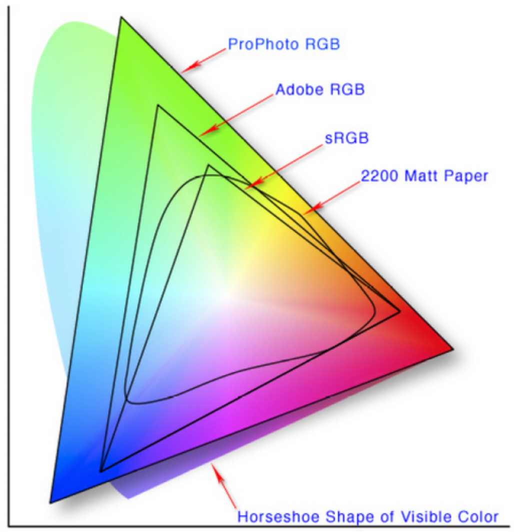


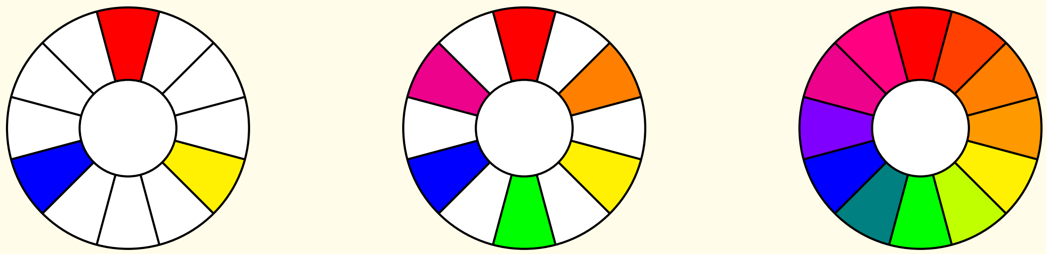

Figure 2: The gamut of visible colors w.r.t. the primary colors compared to gamuts of hardware in monitors. For visual convenience, all gamuts are projected into a 2-dimensional space.

# 1 Hues

Another way to represent this is with color wheels, which is much simpler and gives artists a starting point to test colors.

**Definition 1.1 (Color Wheel)**

A **color wheel** is a visual diagram derived from a set of three primary colors.



(a) Primary (b) Secondary (c) Tertiary

Figure 3: Color wheel progression with primary and secondary colors

**Example 1.1 (Traditional RYB)**

The **RYB** model is a subtractive color model that uses red, yellow, and blue as the primary colors.

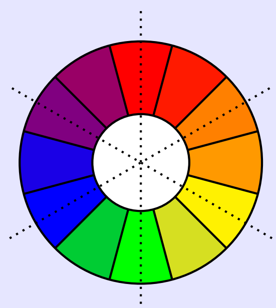


Figure 4: This is the traditional artist's color wheel and represents the way the human eyes perceive color.

**Example 1.2 (RGB Additive)**

The **RGB** model is an additive color model that uses red, green, and blue as the primary colors.

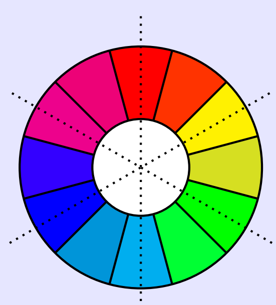


Figure 5: Additive color model used in digital displays.

**Example 1.3 (CMY Subtractive)**

The **CMY** model is a subtractive color model that uses cyan, magenta, and yellow as the primary colors.

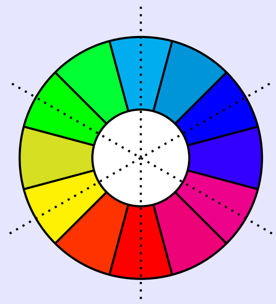


Figure 6: Subtractive color model used in printing.

## 1.1 Analogous Colors

## 1.2 Complementary Colors

The opposite color in the color wheel has the most contrast.

**Definition 1.2 (Complementary Colors)**

Hues $c_1$ and $c_2$ are said to be **complementary hues** with respect if they are on opposite sides of the color wheel.

**Definition 1.3 (Contrast)**

**Contrast** is the difference in color (either in hue, saturation, or brightness) that makes an object visible against a background of different color.

**Theorem 1.1 (Contrast in Complementary Colors)**

Complementary colors maximize contrast.

**Theorem 1.2 (Mixing Complementary Colors)**

Mixing complementary colors
1. in RGB color space results in a white hue.
2. in CMY color space results in a black hue.

## 1.3 Diad Colors

## 1.4 Monochromatic Colors

**Example 1.4 (Monochromatic Blue)**

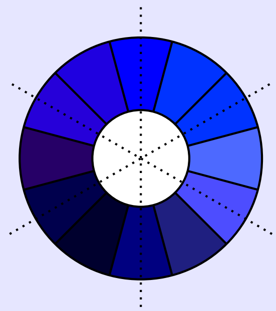


Figure 7: Monochromatic color wheel based on blue variations

## 1.5 Warm Hues

**Example 1.5 (Warm Tones)**

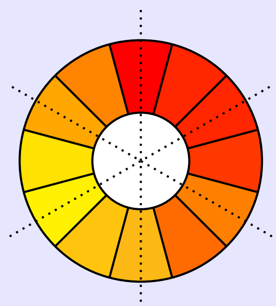


Figure 8: Warm-toned color wheel emphasizing reds, oranges, and yellows

## 1.6 Cool Hues

**Example 1.6 (Cool Tones)**

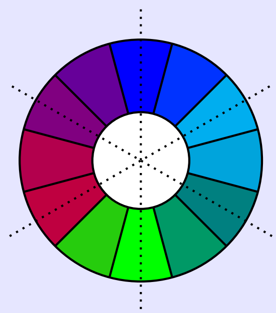


Figure 9: Cool-toned color wheel emphasizing blues, greens, and purples

# 2 Saturation

**Definition 2.1 (Tone)**

The **tone** of a color represents how much gray is in it.

## 2.1 Neon Colors

# 3 Brightness

**Definition 3.1 (Brightness)**

The **brightness** of a color determines how much white or black is in it.

1. The **tint** refers to how much white is in it.
2. The **shade** refers to how much black is in it.

## 3.1 Pastel Colors

## 3.2 Muted Colors